Sundarban Mahavidyalaya

Supplementary Exam. - 2020

B.Sc Part - II General

Sub. - MTMG. Paper - 2nd. F.M - 50

Module - III (25 marks)

Answer Q no. 1 and any five from the rest.

1.(a) Answer any one question 2×1

(i) Write down all the subsets of the set $S = \{p, q, r\}$. What is the power set of the set S?

(ii) Find whether the vectors $(2,4,0)$, $(0,1,0)$ and $(2,6,2)$ are linearly independent in the real vector space R^3.

(b) Answer any one question 3×1

(i) If α, β, γ be the angles which a line makes with the coordinate axes, show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

(ii) Find the equation of the plane passing through the points $(-4, 2, 7)$ and $(1, -3, 5)$ and parallel to z-axis.

2. Find the eigen values of

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$ 4

3. Given the quadratic form $2x_2^2 + 6x_1x_3$, find the
(i) definiteness of the form
(ii) rank, index and signature of the form. 4

4. verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ and hence find A^{-1}! 4

5. Show that $S = \{(x, 2y, 3x) : x, y \text{ are reals}\}$ is a subspace of R^3. Find two basis of S. what is your Conclusion about dimension of S ? 4

6. For any two elements a and b of a group 'G' show that $(ab)^2 = a^2b^2$ if and only if $ab = ba$ in 'G' 4

7. Show that the lines $\frac{x-3}{2} = \frac{y-5}{-3} = \frac{z+3}{-2}$ and $\frac{x-4}{-3} = \frac{y+1}{2} = \frac{z+4}{3}$ are Coplannare. Find the equation of the plane through them. 4

8. Find the equation of the sphere for which the circle $x^2+y^2+z^2+2x-4y+5=0$, $x-2y+3z+1=0$ is a great circle. 4

9. Find the shortest distance between the lines $\frac{x-3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ and $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$
Also find the equation of the line of shortest distance. 4

Module – IV (25 marks)

Answer Q No. 10 and any five from the rest.

10. (a) Answer any one question 2 × 1

(i) State Cauchy's Mean value Theorem

(ii) Evaluate : $\lim_{x \to \pi/2} (1 - \sin x) \tan x$

(b) Answer any one question 3 × 1

(i) Is Rolle's Theorem applicable for the function $f(x) = 1 - x^{2/3}$ in $[-1, 1]$? Justify your answer.

(ii) Evaluate : $\int \sec^4 x \, dx$

11. Find the extreme values of $f(x, y)$ where $f(x, y) = 2x^2 - xy + 2y^2 - 20x$ 4

12. State & Prove Lagrange's Mean value Theorem. 4

13. If $f(x, y) = xy \dfrac{x^2 - y^2}{x^2 + y^2}$ where $x \neq 0$ or $y \neq 0$

$= 0$ where $(x, y) = (0, 0)$

Show that at $(0, 0)$

$$\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$$ 4

14. Determine 'a' and 'b', so that

$$\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$$ 4

15. Test the Convergence of $\int_0^1 x^{n-1} \log x \, dx$ and evaluate its value. 4

16. Prove that

$$\int_0^\infty e^{-x^4} x^2 \, dx \times \int_0^\infty e^{-x^4} \, dx = \frac{\pi}{8\sqrt{2}}$$ 4

17. Evaluate $\int_0^{\pi/2} \int_{\pi/2}^{\pi} e^x \cos(y-x) \, dy \, dx$ 4

18. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$ 4

19. Find the Surface area of the ellipsoid formed by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about it major axis. 4